Handout Week 8

1. **Substitutes.** In class, we discussed one problem with CPI: that people substitute to other goods when the price of one good rises. So CPI overstates the rate of inflation. With that in mind:

   There are two goods in an economy, X and Y. Everybody in the economy has preferences given by: $U = \sqrt{XY}$ and income W. Fix the price of X at 1 (X is dollars). When people have preferences like this, they want to spend half of their money on X and the other half on Y.

   Say that we want to measure inflation as the minimum amount of income a consumer must have to get utility 16.

   (a) Why would this measure be robust to the substitution problem?

   By fixing the level of utility that consumers get from the bundle, we do not care which good the consumers get this utility from, so if consumers substitute to other goods we do not overstate how much more expensive it has become to maintain a fixed standard of living.

   (b) If P is the price of Y and $P = 1$, how much income do we need to get utility 16? What is the bundle of goods that a consumer with this income consumes? Fix this as the CPI market basket.

   Since we know that the consumers will spend half of their income on X and the other half on Y, we have: $X = W/2$ and $Y = W/2$, or $X = Y$. So: $\sqrt{X^2} = 16 \implies X = 16 = Y$. So, to buy this bundle, you will need an income of $16 + 16 = 32$.

   (c) If $P = 4$, how much income do we need?

   Once again, since we know consumers spend half of their income on X and the other half on Y, we have $X = W/2$ and $Y = W/(2 \times 4)$ which tells us that: $X/Y = 4 \implies X = 4Y$. So: $\sqrt{4Y^2} = 16 \implies 2Y = 16 \implies Y = 8$, so $X = 32$, which gives us that: $32 + 4 \times 8 = 64$ is the required income.

   (d) Using the bundle in part (b), compute the rate of inflation that the CPI would give going from part (b) to part (c).

   $16 + 4 \times 16 = 80$ is the value of the market basket in the second period. So, inflation is: $(80 - 32)/32 = 48/32 = 1.5$.

   (e) Compute the rate of inflation using the percentage change in the income required to get utility 16.

   It is given by $(64 - 32)/32 = 1$. Which is less than 1.5. So, CPI overstates the effect of the change in price relative to the utility measure which takes into account

that consumers substitute across goods. What are the challenges for this approach in practice? What are consumer preferences? Don't different people have different preferences? How could we approximate this sort of approach?

2. **Real, Nominal Baseball.** Say, we are interested in comparing the value of the payroll of the New York Mets and the San Diego Padres (American baseball teams) over time. [The CPI and GDP numbers are made up; the baseball numbers are real]

Team	Year	Payroll	Wins	CPI	Nominal GDP	Real GDP
Padres	2013	67 million	76	100	10 trillion	10 trillion
	2012	55 million	76	80	8 trillion	10 trillion
Mets	2013	73 million	74	100	10 trillion	10 trillion
	2012	93 million	74	80	8 trillion	10 trillion

[Fun fact: in Major League Baseball, there are 162 games in a season. There were four teams with exactly the same number of wins in both 2012 and 2013, including the Padres and Mets. The other two were the Minnesota Twins and the Arizona Diamondbacks.]

(a) What is the GDP deflator if we use the same base year as is used in CPI?

The base year for CPI is 2013 since that is the year it is 100. So, the GDP deflator is $\frac{\text{NominalGDP}_{2012}}{\text{RealGDP}_{2012}} = 8/10 = 4/5$.

(b) Calculate the approximate value of the payrolls in base-year dollars using the GDP deflator? The CPI?

Using the GDP deflator: for the Padres in 2012, $\frac{5}{4} \times 55 \approx \frac{5}{4} \times 56 = 70$ million 2013 dollars. For the Mets in 2012, 93 is about 92 (what's a million dollars?), $\frac{5}{4} \times 92 = 115$ million 2013 dollars.

Using the CPI: for the Padres in 2012, since $80/100 = 0.8$, we have $\frac{5}{4} \times 55 \approx 70$ million 2013 dollars. For the Mets in 2012 then, 93 is about 92, $\frac{5}{4} \times 92 = 115$.

(c) What is the rate of inflation from 2012 to 2013 using the CPI? Using the GDP deflator?

Using the CPI: $(100 - 80)/80 = 25\%$

Using the GDP deflator: $(1 - (4/5))/(4/5) = (1/4) = 25\%$.

Note that we do *not* have enough information to switch the base year here. What would we need? We would need information on what Real GDP would be if calculated using 2012 prices.

(d) One of the difficulties with understanding aggregate measures of inflation is that prices do not all rise at the same rate. Say you are a baseball executive and you want a price index that more accurately reflects your interests: say, the WIN index, which is an index of the total amount paid per win in a given year. What is the value of this index in 2013 if the base year is 2012? What is the rate of inflation according to this measure?

To calculate this, the average-price-per-win in 2012 was $(55 + 93)\,/\,(76 + 74) = 148/150$. In 2013 it was $(67 + 73)\,/150 = 140/150$. So, the WIN index in 2013 is: $\frac{140}{148} = \frac{35}{37}$. The rate of deflation is then: $-\frac{2}{37}$.

3. **Inflation.** Every year, a farmer needs to borrow one million dollars to plant his crops, and he pays this loan back later in the year after he has sold the crop at market.

   (a) The farmer borrows money at a 5% interest rate. At the end of the first year, when the farmer must pay back his loan, there is unanticipated inflation of 10%. Who benefits? Assume zero inflation was expected. What is the effective interest rate the farmer actually pays?

   The farmer gets to borrow today's money and pay the lender back with tomorrow's money. If the value of tomorrow's money is unexpectedly low, the farmer is not hurt since he received and used today's money. The lender, however, is paid back in tomorrow's money, and so, is hurt by inflation. To find the effective interest rate: the farmer has to pay back (nominally) 1.05 million dollars. If a current dollar is worth 110% of what it used to be: $1.05 \times \frac{1.00}{1.10} = 0.954....$ So, the effective interest rate is approximately -5.5%. For small percentages like these, an approximation is: Nominal - Inflation = Real.

   (b) It's time to plant again the next year. Will the two parties agree to the same interest rate? Give one reason why they might keep the interest rate the same and another why they might change it.

   It depends on how they interpret the information given by this unanticipated inflation. They will not change the interest rate if they both believe they are correct about the average inflation rate with certainty. Then, they disregard the unanticipated inflation this year as just random chance. If they are uncertain about what the average rate of inflation really is, then they will change what they expect inflation to be. And, so, the interest rate will change.

   (c) Consider the demand curve for loans. If borrowers expect inflation to be high next year, does the demand for loans shift to the left or to the right? What about supply? If lenders expect high inflation what happens to the supply curve?

   If high inflation is expected, then borrowers are willing to take on more debt at higher interest rates (prices) because they believe they will have to pay back less in real terms. So demand shifts to the right. For suppliers, if high inflation is expected, then they are willing to lend out less so supply shift to the left.

   (d) Say the farmer believes that the lenders are far more likely to know what inflation is going to be than he is, so he'd like to know what they expect inflation to be. If he sees the lenders offering a nominal interest rate of 5%, what should he believe about the lender's beliefs about inflation?

   The lenders must believe, at the very least, that inflation is going to be less than 5%.

   (e) Say the market for loans is perfectly competitive. If demand for loans is given by $P = 9 - Q_d$ and supply by $P = 2Q_s$. What is the marginal lender's beliefs about what inflation will be?

   The equilibrium interest rate can be found by solving: $9 - Q = 2Q \implies Q = 3 \implies P = 6$. So the interest rate is 6. We know the marginal lender is the one who is *just* willing to supply the loan so it must be the case that the marginal lender expects inflation to be 6%.

(f) Consider an economy of fortune tellers who can see the future perfectly. Does inflation matter? In other words: should the fortune teller economy try to control inflation?

There is no cost to inflation as far as borrowing and lending go in the fortune teller economy. Since everyone will perfectly anticipate what inflation will be, all inflation will be "priced in" to the nominal interest rate. There is arguably no cost to inflation at all, but you could make an argument that there are costs to updating "menus" if the prices change too often (although if everyone really can see future prices perfectly, there's no reason for menus, right?)